Sample Exam Paper
MAT 7003
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PLEASE ANSWER ALL QUESTIONS on paper provided

The marks for all questions add to 100. If a question is worth X marks then it can be adequately answered in X minutes (or less). Thus, your exam can be answered in 100 minutes (approx) – you have 180 minutes!

You can use: Books, Computer, Web, RODIN…

You cannot work together.

No emails to be exchanged

Question 1 [10 marks]

Sketch the proof that:

The size of the union of any 2 subsets of the set of integers is no bigger than the size of the first subset added to the size of the second subset.

A) Formalise the theorem
B) Give your argument in English
C) Transform your English argument into mathematics
D) Formalise the Proof (of your argument)

Question 2 [15 marks]

You are selecting a tennis partner from a set of candidates. You play against each candidate, in order to establish their relative rank, so that you can choose the best partner.

For example, if there are five players and their true rank is 5, 1, 4, 2, 3, (where 1 is best) then after playing against the first three players you would rank them 3, 1, 2.

As you play each match, you must either accept or reject the candidate immediately before playing the next match. If you do not accept the potential partner after the trial match, the player will choose to partner someone else.

Assume that there are *n candidates and you play trial matches against them in random order:*

•What is the probability you get the best candidate if you play all of the candidates?

•What is the probability if you choose the 1st candidate?

•Assume that you play the first half of the candidates and then continue playing trials until getting a candidate better than any candidate seen so far. Show that you have a better than 25 percent chance of ending up with the best playing partner.

•What is the best strategy for deciding when to stop and accept a candidate that will maximize the probability of getting the best playing partner?

Question 3 [10 marks]

The function f(S,y) gives the number of subsets of size y of a given set S.

For example,

F({a,b,c}, 2) = 3, since the set of subsets of {a,b,c} of size 2 is {{a,b},{a,c},{b,c}}

If S contains m elements, then what value of y will maximize the value of the function f(S,y)?

Prove that your result is correct for any finite set S

Question 4 [10 marks]

Your company manufactures security cameras . These cameras take a photograph (1024 * 1024 pixels) once every second. All photos are then stored on a central server. Unfortunately, this server fills up very quickly and as a consequence the photos need to be backed up every day, which is very expensive.

An external consultant has offered to provide a service to compress the photos so that they take up less storage space without the data compression losing any information.

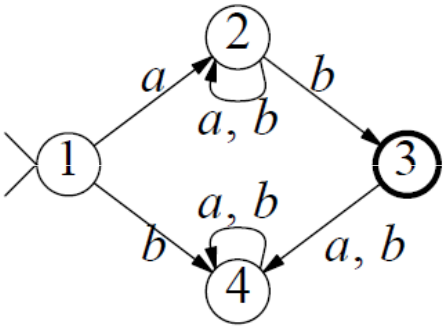
In the contract with the consultant it is written that “the compression function C is guaranteed to reduce the size of any 2 dimensional NxN image without losing information. Thus, through repeated application of C we can reduce the memory needed to store the information in any N*N image by a factor of logN.”

The consultant uses this information to state that buying this service can reduce backup frequency from daily to weekly (i.e by a factor of at least 7)

For what purely mathematical reason(s) should you not buy such a service?

Question 5 [10 marks]

Convert the following Nondeterministic FSA into an equivalent (deterministic) FSA



An NFSA

Question 6 : Turing Machine Design [30 marks]

In a Turing machine, you are free to choose your own data representation on the tape. For example, with problems involving integers one could choose to represent the integers using a number of different bases:

1 – unary
2 – binary
…
10 – decimal
…

A) Design and implement 2 Turing machines for calculating the function « integer division-by-four ». The 2 machines must use 2 different notations. You can choose whatever 2 notations you think simplify the task.

For example, if you chose base 1 and base 10:

- in unary, if we wish to calculate 10/4 we start the tape input with 1111111111 and we finish with output 11,
- in decimal, if we wish to calculate 10/4 we start the tape input with10 and we finish with output 2

B) For each design comment on its computational complexity

QUESTION 7 : Graphs for scheduling [15 marks]

In scheduling final exams, six exams have to be given to seven students. The table below shows the exams for each of the students.

Exams	Amy	Ben	Charles	Debra	Ed	Frank	Georgia
Math	X		X		X		X
Art		X		X		X	
Science	X	X					X
History			X			X	
French					X	X	
Reading	X	X		X	X		X

Find a schedule (allowing some exams to run in parallel) for the exams that is optimal – ie has the minimum number of time slots.

What assumptions did you make in your analysis? Prove optimality of your solution under these assumptions.