A BLIND IDENTIFICATION ALGORITHM ROBUST TO ORDER OVER ESTIMATION

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ABSTRACT

Active research in blind identification of single input multiple output (SIMO) channels has lead to a variety of second order statistics based algorithms, mainly the Subspace and the Linear Prediction approaches. The Subspace algorithm shows good performance, although it requires exact knowledge of the channel order, which is not guaranteed by current order detection algorithms. The Linear Prediction algorithm is sensitive to observation noise while its robustness to channel order over estimation is only theoretical. We propose a new second order statistics based blind channel identification algorithm using a shifted version of the channel output covariance matrix. It proves to be truly robust to channel order over estimation i.e., able to estimate the channel impulse response when the assumed channel order is greater than the exact order and when channel output is corrupted by additive noise and observed over finite time duration. Moreover, the proposed algorithm shows clearly better performance than the Linear Prediction algorithm.

1. INTRODUCTION

Blind identification of communication channels addresses those signal processing techniques that estimate the channel impulse response using solely its output statistics. Such an estimate fed to an equalization algorithm allows restoring, as well as possible, the transmitted data. Second order statistics (SOS) based algorithms are of a particular interest as SOS can be accurately estimated from finite observation samples. Among the more popular, the Subspace (SS) [3] and the Linear Prediction (LP) [5, 1] algorithms. The former achieves better performance but needs exact estimation of the channel order which is a rather delicate and improbable task. The latter is very sensitive to observation noise [2] while its robustness w.r.t. channel order over estimation is only theoretical i.e., does not hold when SOS are only estimated [4].

In this paper, we develop a new algorithm that combines advantages of both algorithms. It shows performance close to that of the SS algorithm while being robust to channel order over estimation.

Vectors are by default in column orientation, while T, H, $*$ stand for transpose, transconjugate and conjugate, respectively. $\mathbf{e}_{k,i}$ is the ith k-dimensional unit vector. $\mathbf{0}_{a,b}$ is the $a \times b$ zero matrix. It is noted $\mathbf{0}$ when its dimension can be inferred from the context. $\|\cdot\|$ denotes the Euclidean norm and $\mathbf{A} \otimes \mathbf{B}$ is the Kronecker product between the matrices $\mathbf{A}$ and $\mathbf{B}$, defined such that its (i,j) block element is $a_{i,j}\mathbf{B}$.

2. SIMO CHANNELS

It is common to model a fractionally spaced and/or multi-sensor receiver by a single input multiple output (SIMO) scheme as depicted in Fig. 1. A set of c filters are driven by a common scalar input $s(n)$. The SIMO channel order m is defined as the maximum among those of the different filters $\mathbf{h}^1, \dots, \mathbf{h}^c$. We define the c-dimensional taps $\mathbf{h}(k) \hat{=} \left[h^1(k) \cdots h^c(k)\right]^T$ ($h^i(k)$ is the k-th tap of the i-th filter) and the SIMO impulse response $\mathbf{h}_m = \left[\mathbf{h}^T(0) \cdots \mathbf{h}^T(m)\right]^T$. The c-dimentional noise corrupted output is $\mathbf{y}(n) \hat{=} \left[y^1(n) \cdots y^c(n)\right]^T$. l successive samples are grouped in $\mathbf{y}_l(n) \hat{=} \left[\mathbf{y}^T(n) \cdots \mathbf{y}^T(n-(l-1))\right]^T$, where l is the *smoothing factor*. The input-output relation is $\mathbf{y}(n) = \mathbf{x}(n) + \mathbf{b}(n) = \mathcal{T}(\mathbf{h}_m)\mathbf{s}_{m+1}(n) + \mathbf{b}(n)$ where $\mathcal{T}(\mathbf{h}_m) \hat{=} [\mathbf{h}(0) \cdots \mathbf{h}(M)]$. When the output is observed over an l symbol period, we have $\mathbf{y}_l(n) = \mathcal{T}_l(\mathbf{h}_m)\mathbf{s}_{l+m}(n) + \mathbf{b}_l(n)$,

where $\mathcal{T}_l(\mathbf{h}_m) \hat{=} \begin{bmatrix} \mathcal{T}(\mathbf{h}_m) \; \mathbf{0} \cdots \mathbf{0} \\ \mathbf{0} \; \mathcal{T}(\mathbf{h}_m) \cdots \mathbf{0} \\ \vdots \\ \mathbf{0} \cdots \mathbf{0} \; \mathcal{T}(\mathbf{h}_m) \end{bmatrix}$ is the $cl \times (l+m)$ *Filtering Matrix* and $\mathbf{0}$ is the c-dimentional null vector.

We define $\mathbf{R}_l \hat{=} \mathrm{E}\left[\mathbf{y}_l(n)\mathbf{y}_l^H(n)\right] = \begin{bmatrix} \gamma(0) & \dots & \gamma(l-1) \\ \vdots & \ddots & \vdots \\ \gamma(1-l) & \dots & \gamma(0) \end{bmatrix}$

and $\mathcal{R}_l \hat{=} \mathrm{E}\left[\mathbf{y}_l(n)\mathbf{y}_l^H(n-1)\right] = \begin{bmatrix} \gamma(1) & \dots & \gamma(l) \\ \vdots & \ddots & \vdots \\ \gamma(-l+2) & \dots & \gamma(1) \end{bmatrix}$.

(For any other process $\mathbf{p}(n)$, we note $\gamma^p(k)$, $\mathbf{R}_l^p$ and $\mathcal{R}_l^p$).
When symbols are decorrelated from noise, we have $\mathbf{R}_l = \mathcal{T}_l(\mathbf{h}_m)\mathbf{R}_{l+m}^s\mathcal{T}_l^H(\mathbf{h}_m) + \mathbf{R}_l^b$ and $\mathcal{R}_l = \mathcal{T}_l(\mathbf{h}_m)\mathcal{R}_{l+m}^s\mathcal{T}_l^H(\mathbf{h}_m) + \mathcal{R}_l^b$. If $s(n)$ is an uncorrelated process, then
$\mathbf{R}_l = \sigma_s^2\, \mathcal{T}_l(\mathbf{h}_m)\mathcal{T}_l^H(\mathbf{h}_m) + \mathbf{R}_l^b$ and
$\mathcal{R}_l = \sigma_s^2\, \mathcal{T}_l(\mathbf{h}_m)\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m) + \mathcal{R}_l^b$ where
$\mathbf{J}_l$ is the $l \times l$ down shifting matrix with 1's on the first subdiag-

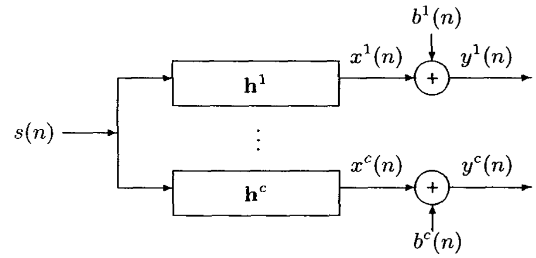


Figure 1: Single input multiple output channel

onal. If, in addition, the noise components are decorrelated, then $\mathbf{R}_l^b = \sigma_b^2\, \mathbf{I}_{cl}$ and $\mathcal{R}_l^b = \sigma_b^2\, (\mathbf{J}_l \otimes \mathbf{I}_c)$. We now recall an important result [6] on the rank of $\mathcal{T}_l(\mathbf{h}_m)$: $\mathcal{T}_l(\mathbf{h}_m)$ is full column rank if

- The transfer functions of the channels $\mathbf{h}^i$, $i = 1, \ldots, c$ have no common zeros in commun, i.e., the polyphase channel $\mathbf{h}_m$ is minimum phase.
- $l \geq \lceil \frac{m}{c-1} \rceil$, i.e., the matrix $\mathcal{T}_l(\mathbf{h}_m)$ is square or tall.

3. A NEW BLIND IDENTIFICATION ALGORITHM

The algorithm assumes knowledge of the correlation matrix $\mathcal{R}_l$, or equivalently of $\mathbf{R}_{l+1}$. The noise power is the smallest eigenvalue of the Hermitian positive definite matrix $\mathbf{R}_{l+1}$ with multiplicity $c(l+1) - (l+m+1)$. We have $\mathcal{R}_l^b = \sigma_b^2\, (\mathbf{J}_l \otimes \mathbf{I}_c)$ and $\mathcal{R}_l - \mathcal{R}_l^b = \sigma_s^2\, \mathcal{T}_l(\mathbf{h}_m)\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m)$.

Hypothesis H 1 $cl \geq l + m$ *i.e.* $l \geq \lceil \frac{m}{c-1} \rceil$.

Under **H**1, $\mathcal{T}_l(\mathbf{h}_m)$ is full column rank ($\mathbf{h}_m$ is assumed to be minimum phase) so that rank $\left(\mathcal{R}_l - \mathcal{R}_l^b\right)$ $=$ rank $\left(\mathcal{T}_l(\mathbf{h}_m)\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m)\right)$ $=$ rank $\left(\mathcal{T}_l(\mathbf{h}_m)\mathbf{J}_{l+m}\right)$ $=$ rank $\left(\mathbf{J}_{l+m}\right) = l + m - 1$. So there exist an orthonormal set $\{\mathbf{n}_{l,1}^{(i)}\}_{i=1,\cdots,w}$ (resp. $\{\mathbf{n}_{l,2}^{(i)}\}_{i=1,\cdots,w}$) of vectors right (resp. left) orthogonal to $\mathcal{R}_l - \mathcal{R}_l^b$, where $w \hat{=} (c-1)l - m + 1$. We have $\mathcal{T}_l(\mathbf{h}_m)\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,1}^{(i)} = \mathbf{0}$. Under **H**1, $\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,1}^{(i)} = \mathbf{0}$ so that $\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,1}^{(i)} = \alpha_1^i \mathbf{e}_{l+m,l+m}$. Similarily, $\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,2}^{(i)} = \alpha_2^i \mathbf{e}_{l+m,1}$. α_1^i and α_2^i are complex constants that can be determined (up to a phase ambiguity) by noting that $\mathbf{n}_{l,j}^{(i)H}\left(\mathbf{R}_l - \mathbf{R}_l^b\right)\mathbf{n}_{l,j}^{(i)} = \mathbf{n}_{l,j}^{(i)H}\mathcal{T}_l(\mathbf{h}_m)\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,j}^{(i)}$ $= \left\|\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,j}^{(i)}\right\|^2 = |\alpha_j^i|^2 \quad , j = 1, 2$. Hence

$$\mathbf{g}_{l-1,l+m}^{(i)} \hat{=} \frac{1}{\sqrt{\mathbf{n}_{l,1}^{(i)H}\left(\mathbf{R}_l - \mathbf{R}_l^b\right)\mathbf{n}_{l,1}^{(i)}}}\mathbf{n}_{l,1}^{(i)*} \quad (1)$$

$$\text{and } \mathbf{g}_{l-1,1}^{(i)} \hat{=} \frac{1}{\sqrt{\mathbf{n}_{l,2}^{(i)H}\left(\mathbf{R}_l - \mathbf{R}_l^b\right)\mathbf{n}_{l,2}^{(i)}}}\mathbf{n}_{l,2}^{(i)*} \quad (2)$$

are $(l-1)$ order, maximum (resp. minimum) delay Zero Forcing (ZF) equalizers, for $i = 1, \cdots, w$. The channel taps can be retrieved from the output statistics and any among the ZF equalizers since $\mathbf{h}(k) = \mathrm{E}\left(\mathbf{y}(n)s(n-k)^*\right) = \mathrm{E}\left(\mathbf{x}(n)\mathbf{x}_l^H(n-k)\right)\mathbf{g}_{l-1,1}^{(i)*}$ $= \mathrm{E}\left(\mathbf{x}(n)\mathbf{x}_l^H(n-k+l+m-1)\right)\mathbf{g}_{l-1,l+m}^{(i)*}$ which can be written as:

Based on $\mathbf{g}_{l-1,1}^{(i)}$, the channel response is

$$\mathbf{h}_m = \begin{bmatrix} \gamma^x(0) & \gamma^x(1) & \cdots & \gamma^x(l-1) \\ \gamma^x(1) & \gamma^x(2) & & \gamma^x(l) \\ \vdots & & & \vdots \\ \gamma^x(m) & \cdots & & \gamma^x(l+m-1) \end{bmatrix} \mathbf{g}_{l-1,1}^{(i)*} \quad (3)$$

Based on $\mathbf{g}_{l-1,l+m}^{(i)}$, the channel response is

$$\mathbf{h}_m = \begin{bmatrix} \gamma^x(-l-m+1) & \cdots & \gamma^x(-m) \\ \gamma^x(-l-m+2) & & \gamma^x(-m+1) \\ \vdots & & \vdots \\ \gamma^x(-l+1) & \cdots & \gamma^x(0) \end{bmatrix} \mathbf{g}_{l-1,l+m}^{(i)*} \quad (4)$$

As the equalizers are determined to within a phase ambiguity, the channel response is determined up to a phase rotation as well. Relations (3) and (4) require knowledge of the so far missing non zero terms $\gamma(l), \ldots, \gamma(m)$. We hence strengthen **H1** :

Hypothesis H 2 $l \geq m$.

4. ROBUSTNESS TO ORDER OVER ESTIMATION

We now prove an important feature of the proposed algorithm which is its ability to estimate the exact channel impulse response $\mathbf{h}_m$ when the channel order is over estimated. If we detect an order $m' > m$, $\mathcal{R}_l - \mathcal{R}_l^b$ rank is estimated to be $l + m' + 1$.

Any among the vector $\mathbf{n}_{l,1}^{(i)}$ and $\mathbf{n}_{l,2}^{(i)}$, $i = 1, \cdots, w$, suffices to get the channel response following the steps above. If $\mathbf{g}_{l-1,l+m'}^{(i)}$ and $\mathbf{g}_{l-1,1}^{(i)}$ are constructed as mentioned above, the algorithm attempts to compute either using (3)

$$\begin{bmatrix} \gamma^x(0) & \gamma^x(1) & \cdots & \gamma^x(l-1) \\ \gamma^x(1) & \gamma^x(2) & & \gamma^x(l) \\ \vdots & & & \vdots \\ \gamma^x(m) & \cdots & & \gamma^x(l+m-1) \\ \vdots & & & \vdots \\ \gamma^x(m') & \cdots & & \gamma^x(l+m'-1) \end{bmatrix} \mathbf{g}_{l-1,1}^{(i)*} = \begin{bmatrix} \mathbf{h}_m \\ \mathbf{0} \end{bmatrix}$$

or (using (4))

$$\begin{bmatrix} \gamma^x(-l-m'+1) & \cdots & \gamma^x(-m') \\ \vdots & & \vdots \\ \gamma^x(-l-m) & \cdots & \gamma^x(-m-1) \\ \gamma^x(-l-m+1) & \cdots & \gamma^x(-m) \\ \gamma^x(-l-m+2) & & \gamma^x(-m+1) \\ \vdots & & \vdots \\ \gamma^x(-l+1) & \cdots & \gamma^x(0) \end{bmatrix} \mathbf{g}_{l-1,l+m'}^{(i)*} = \begin{bmatrix} \mathbf{0} \\ \mathbf{h}_m \end{bmatrix}$$

where we have used the fact that, because $\mathbf{x}(n)$ is an m-order moving average process, $\gamma^x(k) = \mathbf{0}$, if $|k| > m$. Consequently, the so estimated channel response is a zero padded version of the channel response, and hence, can be used for equalization purposes.

5. THE ESTIMATED SOS CASE

Because of the finite sample size, the estimate of $\mathcal{R}_l$ will not be rank deficient. The vectors $\mathbf{n}_{l,1}^{(i)}$ (resp. $\mathbf{n}_{l,2}^{(i)}$) are chosen to be those right (resp., left) singular vectors associated with the w smallest singular values of the estimate of $\mathcal{R}_l$. Each of these vectors leads to an estimate of the channel response. We should therefore introduce a criterion to choose among the $2w$ estimates $\{\hat{\mathbf{h}}_m^{(i)}\}$, $i = 1, \cdots, 2w$. We propose the following criterion

$$\hat{\mathbf{h}}_m = \mathrm{argmin}_i \left\|\mathcal{T}_l(\hat{\mathbf{h}}_m^{(i)})\mathcal{T}_l^H(\hat{\mathbf{h}}_m^{(i)}) + \widehat{\sigma_b^2}\,\mathbf{I}_{cl} - \hat{\mathbf{R}}_l\right\|$$

Finally, we can enhance the algorithm's performance if the channel response can be estimated up to a complex constant i.e., allowing for a phase and amplitude ambiguity. We modify (1) and (2) and compute $\mathbf{g}_{l-1,l+m}^{(i)} \hat{=} \mathbf{n}_{l,1}^{(i)*}$ and/or $\mathbf{g}_{l-1,1}^{(i)} \hat{=} \mathbf{n}_{l,2}^{(i)*}$.

W.r.t. these modifications, the algorithm can be better written as follows:

- Estimate an order m not inferior to the true channel order.
- Choose a smoothing factor $l \geq m$.

- Compute the estimate $\hat{\mathcal{R}}_l$ of $\mathcal{R}_l$.
- Estimate noise power $\widehat{\sigma_b^2}$ as the average of the $(c-1)(l+1)-m$ smallest eigenvalues of $\hat{\mathbf{R}}_{l+1}$.
- Compute the cl-dimensional left singular vectors $\mathbf{n}_{l,1}^{(i)}$ and right singular vectors $\mathbf{n}_{l,2}^{(i)}$ associated with the $w \hat{=} (c-1)l - m + 1$ lowest singular values of $\hat{\mathcal{R}}_l - \widehat{\sigma_b^2}\,(\mathbf{J}_l \otimes \mathbf{I}_c)$. Built the set $\{\mathbf{n}_l^{(i)}\} = \{\mathbf{n}_{l,1}^{(i)}\} \cup \{\mathbf{n}_{l,2}^{(i)}\}$.
- For each $\mathbf{n}_l^{(i)}$, estimate the ZF equalizer $\mathbf{g}_{l-1}^{(i)} = \frac{1}{\sqrt{\mathbf{n}_l^{(i)H}\left(\hat{\mathbf{R}}_l - \widehat{\sigma_b^2}\,\mathbf{I}_{cl}\right)\mathbf{n}_l^{(i)}}}\mathbf{n}_l^{(i)*}$ (phase ambiguity) or $\mathbf{g}_{l-1}^{(i)} = \mathbf{n}_l^{(i)*}$ (phase and amplitude ambiguity).
- For each $\mathbf{g}_{l-1}^{(i)}$, estimate the corresponding $\hat{\mathbf{h}}_m^{(i)}$ using (3) or (4) depending on whether $\mathbf{n}_l^{(i)}$ is a left or right singular vector, respectively.
- Choose $\hat{\mathbf{h}}_m = \text{argmin}_{i \leq 2w} \left\| \mathcal{T}_l(\hat{\mathbf{h}}_m^{(i)})\mathcal{T}_l^H(\hat{\mathbf{h}}_m^{(i)}) + \widehat{\sigma_b^2}\,\mathbf{I}_{cl} - \hat{\mathbf{R}}_l \right\|$.

6. COMPARISON WITH EXISTING ALGORITHMS

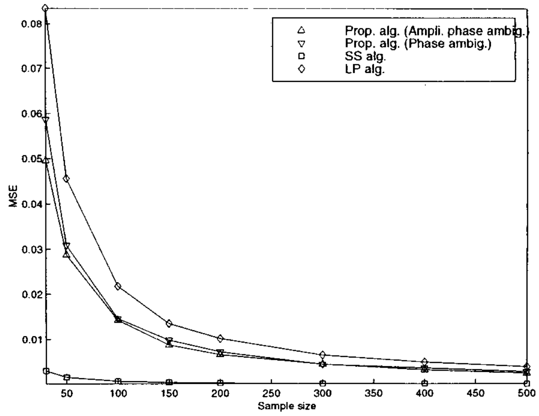
So far, the SS algorithm and the LP algorithm are the most important SOS based blind equalization algorithms, the former because of its good performance and the latter because of its robustness (even though purely theoretical) to channel order over estimation. The proposed algorithm shows to be an interesting alternative to both approaches. It combines advantages of both algorithms and moreover exhibits the very practical feature of being truly robust to channel order over estimation i.e., the channel impulse response still can be estimated when the channel order is over estimated and only finite noise corrupted observation samples are available.

As it will be shown through simulations in section 7, the proposed algorithm shows performance intermediate between SS and LP algorithms. This can be justified as follows. Unlike the SS algorithm, the noise energy needs to be estimated which may lead into additional estimation error. The better performance of the proposed algorithm compared to the LP algorithm can be justified in different ways. First, the proposed algorithm uses a singular vector search, in a way similar to the SS algorithm, and hence avoids explicit inversion of the correlation matrix, as does the LP algorithm, which is numerically risky when the matrix is poorly conditioned. Second, like the LP algorithm, the proposed algorithm estimates a ZF equalizer prior to channel response estimation. However, this ZF equalizer is shorter (when $l = m$) than that estimated by the LP algorithm and, for that reason, estimation error is expected to be lower.

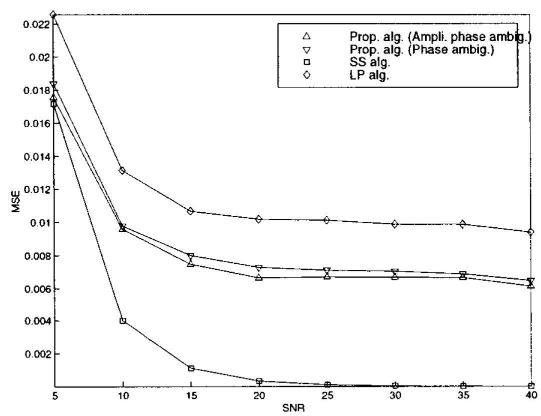
One further advantage of the proposed algorithm over the SS algorithm is that the channel response is estimated up to a rotation, a feature shared by the LP algorithm.

7. SIMULATIONS

A series of simulations has been conducted to test the proposed algorithm w.r.t. to different observation conditions (noise power, sample size), algorithm parameters (smoothing factor, ambiguity) and



(a) $SNR = 20dB$.



(b) Sample size= 200.

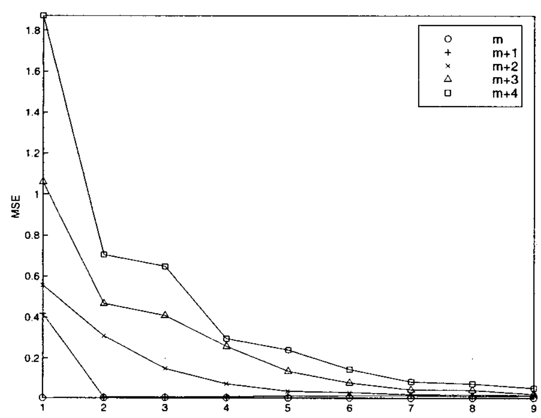
Figure 2: Algorithms comparison. $w = 2$ (for the proposed algorithm).

in comparison with existing SOS based blind identification algorithms, namely, the SS and the LP algorithms.

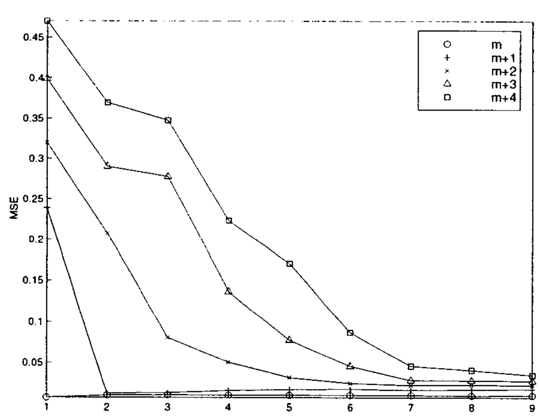
We tested the proposed algorithm with an AWGN SIMO channel with 2 subchannels whose impulse responses are given by $\mathbf{h}^1 = [-0.6804\ \ 0.177\ \ -0.0902]$ and $\mathbf{h}^2 = [0.4281\ \ -0.2446\ \ -0.5043]$ so that $\|\mathbf{h}_m\| = 1$. We used a binary source with equiprobable i.i.d. symbols. 1000 Monte Carlo simulations were conducted. Eachtime, phase (and amplitude when applicable) ambiguity is "removed" from the channel estimate. The algorithm performance was expressed in terms of the mean square error (MSE) given by $\min_{p \geq 0, q \geq 0, p+q=m'-m} \left\| \hat{\mathbf{h}}_{m'} - \begin{bmatrix} \mathbf{0}_{p,1} \\ \mathbf{h}_m \\ \mathbf{0}_{q,1} \end{bmatrix} \right\|^2$ averaged over the Monte Carlo realizations. m' is the channel estimate ($\hat{\mathbf{h}}_{m'}$) chosen order, superior to m as channel order over estimation is considered.

Fig.2 compares the proposed algorithm (with phase and amplitude/phase ambiguity) to the SS and LP algorithms w.r.t. the sample size and channel noise. The exact channel order is assumed to be detected. It shows that the proposed algorithm clearly outperforms the LP algorithm but is less efficient than the SS algorithm.

The more important issue of channel order over estimation is viewed in Fig.3 for both phase and phase-amplitude ambiguity versions of the proposed algorithm. It demonstrates that the proposed algorithm is able to identify the channel impulse response from finite sample size when the detected channel order is higher than its true order. Neither SS nor LP algorithms was able to achieve identification in such conditions. The figures show also that the estimation error can be reduced by increasing the parameter w. This is particularily needed when only a phase ambiguity is tolerated.



(a) Phase ambiguity



(b) Phase and amplitude ambiguity

Figure 3: Channel order over estimation. The legend shows the assumed channel order m'. Sample size= 200, $SNR = 20dB$.

8. CONCLUSION

We have proposed a new second order statistics based blind identification algorithm that is truly robust to channel order over estimation. It is able to estimate the channel response when only an over estimated value of the channel order is detected and when only a finite number of noise corrupted observation samples is available. This is qualified as 'true robustness' in comparison to the LP algorithm for which such estimation is possible under the (unpractical) hypothesis of second order statistics being exactly known. The SS algorithm is known to be inefficient when detected channel order is wrong. In addition, the proposed algorithm behaves better than the LP algorithm, especially when the channel response is allowed to be estimated up to a constant.

9. REFERENCES

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